BAREM - cls. a V-a

1. $N = 10\,000\,\overline{ab} + 100\,\overline{cd} + \overline{ef} =$
$= \overline{ef} + \overline{cd} \cdot (7 \cdot 14 + 2) + \overline{ab} \cdot (7 \cdot 1428 + 4) =$
$= M7 + \underbrace{\overline{ef} + 2\overline{cd} + 4\overline{ab}}_{M7} = M7 \vdots 7$

Acum $N \vdots 7$ dacă și numai dacă $(\overline{ef} + 2\overline{cd} + 4\overline{ab}) \vdots 7$

2. $\Leftrightarrow c \cdot (10a + b) - b(10a + c) = 10 \Leftrightarrow$
$10ac + \cancel{bc} - 10ab - \cancel{bc} = 10 \Leftrightarrow 10a(c - b) = 10 \Leftrightarrow$
$a(c - b) = 1 \Leftrightarrow a = 1$ și $c - b = 1 \Leftrightarrow$
$a = 1$ și $c = b + 1$ unde $b \in \{0, 1, 2, \ldots, 8\}$
Obținem $\overline{ab} = \{10, 11, \ldots, 18\}$
$\overline{ac} = \{11, 12, \ldots 19\}$
$\overline{ab} \cdot \overline{ac} = \{110, 132, 156, \ldots, 342\}$
Suma cifrelor $= \{2, 6, 12, 11, 3, 6, \ldots, 9\}$
Soluția: $\overline{ab} = 12$, $\overline{ac} = 13$

3. $x + y = 2015$
$x - y = 403k \Rightarrow x = y + 403k \Rightarrow 2y + 403k = 2015$
$\Leftrightarrow 2y = 2015 - 403k \Rightarrow k = 2p + 1 = \{1, 3, 5\}$
$k = 1 \Rightarrow \ldots$ $y = 806$; $x = 1209$
$k = 3$ $y = 403$; $x = 1612$
$k = 5$ $y = 0$ $x = 2015$
$k = 7$ (F)

4. a) $\frac{1}{x} > \frac{1}{x+y}$, $\frac{1}{y} > \frac{1}{x+y}$ $\overset{\oplus}{\Longrightarrow} \frac{1}{x} + \frac{1}{y} > \frac{2}{x+y}$

b) aplicăm (a) $\Rightarrow$

$\frac{1}{501} + \frac{1}{1514} > \frac{2}{2015}$
$\frac{1}{502} + \frac{1}{1513} > \frac{2}{2015}$
$\ldots$
$\frac{1}{1007} + \frac{1}{1008} > \frac{2}{2015}$

$\overset{\oplus}{\Longrightarrow} S > \frac{2 \cdot 507}{2015} = \frac{1014}{2015} > \frac{1}{2}$ (A)

BAREM - cls VI-a

1. Împărțim $\mathbb{N}$ în 5 submulțimi de forme generale $5K$, $5K+1$, $5K+2$, $5K+3$, $5K+4$ și obținem:

$n=5K \Rightarrow n+25=5K+25=5(K+5)$ compus

$n=5K+1 \Rightarrow n+49=5K+50=5(K+10)$ — „ —

$n=5K+2 \Rightarrow n+13=5K+15=5(K+3)$ — „ — $\quad K\neq 0$

$n=5K+3 \Rightarrow n+37=5K+40=5(K+8)$ — „ —

$n=5K+4 \Rightarrow n+1=5(K+1)$ — „ —

Dacă $K=0 \Rightarrow n+1=5K+4+1=5$

$n+13=5K+4+13=17$

$n+25=5K+4+25=29 \quad (A) \Rightarrow n=4$

$n+37=5K+4+37=41$

$n+49=5K+4+49=53$

2. a) $1\in M \Rightarrow 3\in M \Rightarrow 9\in M \Rightarrow 27\in M \Rightarrow 81\in M \Leftrightarrow (5\cdot 17-4)\in M$

$\Rightarrow 17\in M \Rightarrow 51\in M \Leftrightarrow (5\cdot 11-4)\in M \Rightarrow 11\in M$

b) Presup. $5c+2\in M \Rightarrow 3(5c+2)\in M$ adică $15c+6\in M$

$\Leftrightarrow 5(3c+2)-4\in M \Rightarrow 3c+2\in M$

c) $373\in M \Rightarrow 3\cdot 373\in M \Leftrightarrow 1119\in M \Rightarrow 3\cdot 1119\in M \Leftrightarrow 3357\in M$

$3357=3355+2=5\cdot 671+2\in M \overset{b)}{\Rightarrow} 3\cdot 671+2\in M \Leftrightarrow 2013+2=2015\in M$

3. a) $\hat{A}+\hat{C}=90^\circ \Rightarrow \hat{A}=60^\circ, \hat{C}=30^\circ \Rightarrow \widehat{DAB}=\widehat{DAC}=30^\circ$

În ΔABD: BO med. $\Rightarrow BO=AO=DO$ și $\widehat{ADB}=60^\circ \Rightarrow \Delta BOD$ echilateral

$\Rightarrow \widehat{AON}=\widehat{BOD}=60^\circ$ și $\widehat{DAC}=30^\circ \Rightarrow \widehat{ANB}=90^\circ \Rightarrow BO\perp AC$

b) ΔACD isoscel $\Rightarrow$ med. DM este și înălțime $\Rightarrow DM\perp AC$

$\Rightarrow BO \parallel DM$ ($\measuredangle$ corresp. $\equiv$)

c) ΔADM: ON l. mij. $\Rightarrow AN=\dfrac{AM}{2}$, $AM=\dfrac{AC}{2}$ $\Big| \Rightarrow AN=\dfrac{1}{4}\cdot AC$.

4. Construim ΔBEC echilateral a.î. A și E să fie de o parte și de cealaltă parte a dr. BC.

$\Delta ABE \equiv \Delta ACE \equiv \Delta ACD$ (L.U.L.) $\Rightarrow \measuredangle^{le} = 10^\circ, 30^\circ, 140^\circ$.

Clasa a VII-a — BAREM

1. Aplic. ineg. mediei $\frac{(k+1)+k}{2} \geq \sqrt{k(k+1)} \iff \frac{2}{2k+1} \leq \frac{1}{\sqrt{k(k+1)}}$

$\iff \frac{1}{(2k+1)\sqrt{k(k+1)}} \leq \frac{1}{2k(k+1)} = \frac{1}{2}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

$\Rightarrow \frac{1}{3\sqrt{2}} \leq \frac{1}{2}\left(\frac{1}{1}-\frac{1}{2}\right)$

$\frac{1}{5\sqrt{6}} \leq \frac{1}{2}\left(\frac{1}{2}-\frac{1}{3}\right)$

$\dots$

$\frac{1}{(2n+1)\sqrt{n(n+1)}} \leq \frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

$\overset{(+)}{\Rightarrow} S \leq \frac{1}{2}\left(1-\frac{1}{n+1}\right) = \frac{1}{2}\cdot\frac{n}{n+1} \iff$

$S < \frac{n}{2n+2}$

2. Fie cele 3 nr.: $x-1,\ x,\ x+1,\ x \in \mathbb{N}^* \Rightarrow S = 3x$

3 cazurile $3x = x(x-1) = x^2-x \iff x^2-4x=0 \iff x(x-4)=0 \Rightarrow x=4$

$3x = x(x+1) = x^2+x \quad x^2-2x=0 \quad x(x-2)=0 \Rightarrow x=2$

$3x = (x-1)\cdot(x+1) = x^2-1 \quad x^2-3x=1 \iff x(x-3)=1$ F.

$S: \{(3,4,5);\ (1,2,3)\}$

3. Din a) $\Rightarrow \mathcal{A}_{\Delta ABD} = \mathcal{A}_{\Delta ADC} \iff AD\cdot h_B = AD\cdot h_C \Rightarrow d(B,AD) = d(C,A$ $\Rightarrow BC \parallel AD$.

b) $\Rightarrow \mathcal{A}_{\Delta BCD} = \mathcal{A}_{\Delta ACD} \iff \dots \Rightarrow d(B,DC) = d(A,DC) \Rightarrow AB \parallel DC$

$\Rightarrow ABCD$ paralelogram.

4. Se consideră triunghiul echilateral BCE cu A și E de o part și de alta a dreptei BC, atunci $\Delta ACE \equiv \Delta ACD$ (LUL) $\Rightarrow \widehat{ADC} \equiv \widehat{AEC}$

dar $\Delta ACE \equiv \Delta ABE$ (LUL) $\Rightarrow \widehat{AEC} = 30^\circ \Rightarrow \widehat{ADC} = 30^\circ \Rightarrow \widehat{DAC} = 45^\circ$

dar și $\widehat{ACB} = 45^\circ \Rightarrow AD \parallel BC$ iar AB nu este paralelă cu CD

$(\widehat{BAC} = 90^\circ;\ \widehat{ACD} = 105^\circ)$, am demonstrat $ABCD$ este trapez

Clasa a VIII-a BAREM

1. $\frac{x^2+x+7}{x^2+x+1}=\frac{(x^2+x+1)+6}{x^2+x+1}=1+\frac{6}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}<1+\frac{6}{\frac{3}{4}}=9$.

$\frac{270x^2+90x+21}{18x^2+6x+2}=\frac{9(18x^2+6x+2)+3(36x^2+12x+1)}{18x^2+6x+2}=9+\frac{3(6x+1)^2}{2\left[\left(3x+\frac{1}{2}\right)^2+\frac{3}{4}\right]}>9$.

2) $\Leftrightarrow 2a^2+2b^2-4ab=a^2+b^2+2ab+2a+2b+1 \Rightarrow$
$2(a-b)^2=(a+b+1)^2$. Dacă $a\neq b \Rightarrow 2=\left(\frac{a+b+1}{a-b}\right)^2$ şi cum $\frac{a+b+1}{a-b}\in\mathbb{Q}$
$\Rightarrow \pm\sqrt{2}\in\mathbb{Q}$ (F) $\Rightarrow a=b \Rightarrow a+b+1=0 \Rightarrow a=b=-\frac{1}{2}$
Sau se poate vedea egalitatea ca ec. gr. II în a sau b...

3) $\Rightarrow \frac{a\cdot b}{3}=\frac{b\cdot c}{5}=\frac{a\cdot c}{15}=\frac{ab+bc+ac}{3+5+15}=\frac{\frac{1}{2}\cdot A_t}{23}=4 \Rightarrow$

$a\cdot b=4\cdot 3=12$
$b\cdot c=4\cdot 5=20$
$a\cdot c=4\cdot 15=60$
$\Rightarrow a^2b^2c^2=14400 \Rightarrow a\cdot b\cdot c=120 \Rightarrow V=120\,cm^3$

Din $a\cdot b\cdot c=120 \wedge a\cdot b=12 \Rightarrow c=10 \dots \Rightarrow a=6 \Rightarrow b=2$
$d=\sqrt{\quad}=\dots=2\sqrt{35}$ (cm)

4) Fie $\frac{BN}{BR}=r_1$, $\frac{BP}{BS}=r_2$, unde $BM\cap VD=\{R\}\wedge BP\cap CD=\{S\}$
Dacă $r_1=r_2 \Rightarrow PN\parallel SR$ şi cum $SR\subset(VCD)\Rightarrow PN\parallel(VCD)$
Acum $\Delta CPS\sim\Delta APB \Rightarrow \frac{CP}{AP}=\frac{CS}{AB}=\frac{PS}{PB}\Rightarrow\frac{CP}{CP+AP}=\frac{PS}{PS+PB}\Leftrightarrow$
$\frac{CP}{AC}=\frac{PS}{BS}\Rightarrow\frac{CP}{AC}=1-\frac{BP}{BS}\Rightarrow\frac{BP}{BS}=1-\frac{CP}{AC}$, adică $\frac{AP}{AC}=r_2$
Pentru r_1, luăm T. lui Menelaus:
ΔVDO: $R-M-B$: $\frac{RV}{RD}\cdot\frac{MO}{MV}\cdot\frac{BD}{BO}=1\Rightarrow\frac{RV}{RD}=\frac{1}{2}$ şi apoi

$\frac{RV}{RV+RD}=\frac{1}{1+2}\Leftrightarrow\frac{RV}{VD}=\frac{1}{3}$
ΔRDV: $V-M-O$: $\frac{VR}{VD}\cdot\frac{MB}{MR}\cdot\frac{OD}{OB}=1\Rightarrow\frac{MB}{MR}=3\Rightarrow\frac{MB}{MB+MR}=\frac{3}{3+1}$
$\Rightarrow\frac{NB}{MB}=\frac{4}{3}r_1$ din $\left(\frac{MB}{RB}=\frac{3}{4}\right.$ şi $\left.\frac{NB}{BR}=r_1\right)$
adică $r_1=\frac{3}{4}\cdot\frac{NB}{MB}$, iar $r_2=\frac{AP}{AC}$, cum din ip. avem
$\frac{3}{4}\cdot\frac{BN}{BM}=\frac{AP}{AC}\Rightarrow r_1=r_2\Rightarrow PN\parallel(VDC)$